The Use of Vector Fields To Model the Physical Blockage From Power Supply, Cable, and Transformer Sources

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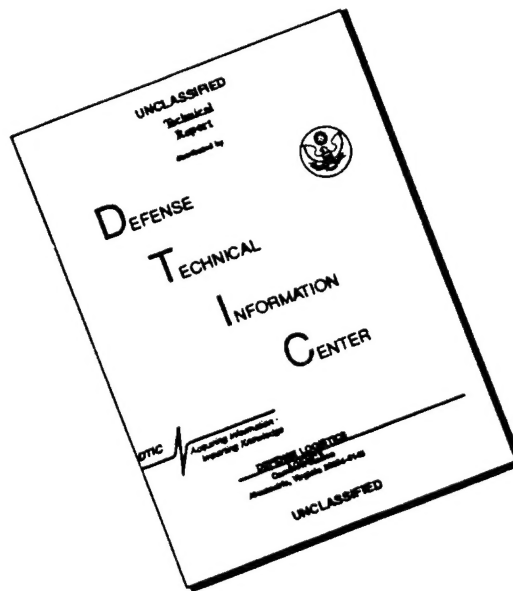### Newport, Rhode Island

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Preface

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This effort was designed to improve electromagnetic (EM) analysis techniques that can be used to supplement the Intelligent EMC Analysis and Design System (IEMCADS). The primary objective of the work was to model the physical blockage of magnetic fields produced by IEMCADS sources. The sources include transformers, cables, and power supplies, all of which IEMCADS represents in a simplistic and scalar form to yield worst-case field conditions. These sources do not give rise to uniform fields or plane waves, but to fields that vary as inverse powers of the radial distance. It was decided that the best way to obtain an analytical solution to the problem was to expand the IEMCADS sources in Legendre polynomials and use a hollow cylindrical physical blockage as a representation of cabinets. This technique enables the fields in the various regions around and within the physical blockage to be modeled as expansions in Legendre polynomials. These expansions make the source modeling compatible with the physical blockage modeling, and thus allow boundary conditions to be applied for the B and H fields at the two interfaces of the cylindrical shell. The application of the boundary conditions generates a system of 10 equations and 10 unknowns for the coefficients of expansion for each source type. The solution of the 10 equations and 10 unknowns yields the coefficients of expansion of the Legendre polynomials. The final Legendre polynomial expansions produce the magnetic scalar potentials for the different sources. The gradients of the magnetic scalar potentials are taken to yield the vector B and H fields in the regions of the source, the regions outside and within the shell of the physical blockage, and the hollow region of the physical blockage.

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THE USE OF VECTOR FIELDS TO MODEL THE PHYSICAL BLOCKAGE FROM POWER SUPPLY, CABLE, AND TRANSFORMER SOURCES

INTRODUCTION

This effort was designed to improve the electromagnetic (EM) analysis techniques that can be used to supplement the Intelligent EMC Analysis and Design System (IEMCADS)[1] developed at the Electromagnetic Compatibility (EMC) Branch of the Naval Undersea Warfare Center Detachment in New London, Connecticut. The primary objective of this work was to model the physical blockage of magnetic fields produced by the IEMCADS sources.

The IEMCADS uses such sources as transformers, cables, and power supplies, which it represents in a simplistic and scalar form to yield worst-case magnetic field conditions. These sources do not give rise to uniform fields or plane waves, but to fields that vary as inverse powers of the radial distance.

It was decided that the best way to obtain an analytical solution to this problem was to expand the sources that exist in IEMCADS in Legendre polynomials and use a hollow cylindrical physical blockage as a representation of cabinets. This technique enables the fields in the various regions around and within the physical blockage to be modeled as expansions in Legendre polynomials.[2-6] These expansions make the source modeling compatible with the physical blockage modeling, and thus boundary conditions can be applied for the B and H fields* at the two interfaces of the cylindrical shell. The application of the boundary conditions generates a system of 10 equations and 10 unknowns for the coefficients of expansion for each source type. The solution of the 10 equations and 10 unknowns yields the coefficients of expansion of the Legendre polynomials. The final Legendre polynomial expansions produce the magnetic scalar potentials for the different sources. The gradients of the magnetic scalar potentials are then taken to yield the vector B and H fields in the regions of the source, the regions outside and within the shell of the physical blockage, and the hollow region of the physical blockage.

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* According to Feynman's definition, B represents the magnetic field vector and H represents the magnetizing field.
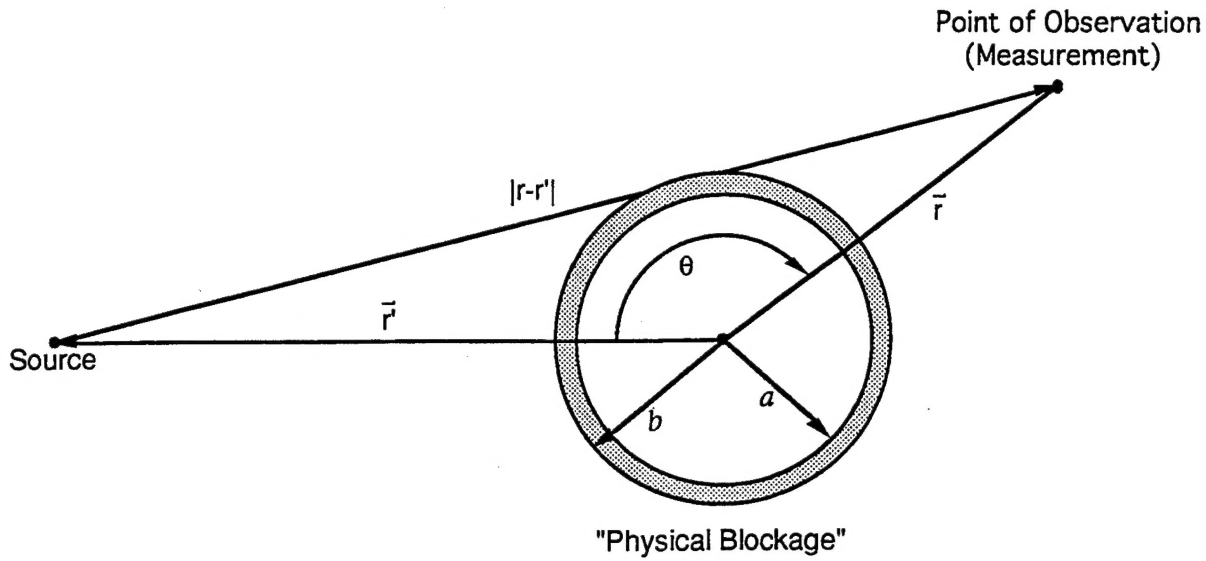
Figure 1. Representation of the Physical Blockage

THE MATHEMATICAL MODELS

The IEMCADS power supply field varies as $\frac{1}{|\vec{r}-\vec{r}'|}$ and the IEMCADS transformer varies as $\frac{1}{|\vec{r}-\vec{r}'|^3}$. From figure 1, $\vec{r}'$ is the distance from the source to the center of the cylindrical shell and $\vec{r}$ is the distance from the cylindrical axis to the point of observation. The coordinates of the point of observation (experimental point) are always $\vec{r}$ and θ.

GENERAL METHOD FOR APPLICATION OF THE BOUNDARY CONDITIONS

The coefficients of the Legendre polynomial expansions[4] are solved by application of the boundary conditions at the two interfaces: $r = a$ and $r = b$. These boundary conditions are the continuity of the tangential components of the H field and the normal component of the B field. To apply these boundary conditions requires that the coefficients of $(\text{cosine}^n)(\text{sine}^m)$, where n = 0,1,2 and m = 0,1,2, be set equal to zero.

The application of the boundary conditions involves two cases: $\vec{r} \geq \vec{r}'$ and $\vec{r} \leq \vec{r}'$. The coefficients of expansion of the Legendre polynomials are different for these two cases since the sources are expanded as a binomial expansion in powers of $\frac{r'}{r}$ for the case $r \geq r'$ and in powers of $\frac{r}{r'}$ for the case $r \leq r'$. (Physically, the case $r \geq r'$ means that the source is closer to the physical blockage than the point of observation and

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the case $r' \geq r$ means that the point of observation is closer to the physical blockage than the source.) The binomial expansions of the sources are necessary to obtain coefficients of powers of sines and cosines. These expansions can then be combined with the Legendre polynomial expansions to generate the 10 equations and 10 unknowns. The 10 equations yield unique solutions for the 10 unknowns, which are then substituted back into equations (1), (2), and (3) (see below). The gradient operations were carried out in these three equations to yield the general solutions.

THE MATHEMATICAL FORM FOR THE $\frac{1}{|\vec{r} - \vec{r}'|}$ SOURCE

The H fields are given in three regions: $r > b$, $a < r < b$, and $r < a$. For $r > b$, the H field is given by

$$\vec{H} = \frac{B_o}{|\vec{r} - \vec{r}'|} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|} - \nabla \sum_{\ell=0}^{2} \frac{\alpha_\ell}{r^{\ell+1}} P_\ell \left(\cos(\theta) \right). \tag{1}$$

For $a < r < b$,

$$\vec{H} = -\nabla \sum_{\ell=0}^{2} \left(\beta_\ell r^\ell + \frac{\gamma_\ell}{r^{\ell+1}} P_\ell \left(\cos(\theta) \right) \right), \tag{2}$$

and for $r < a$,

$$H = -\nabla \sum_{\ell=0}^{2} \delta_\ell r^\ell P_\ell \left(\cos(\theta) \right). \tag{3}$$

THE $\frac{1}{|\vec{r} - \vec{r}'|}$ SOURCE IN COMPONENT FORM AND ITS BINOMIAL EXPANSION

The B field for the $\frac{1}{|\vec{r} - \vec{r}'|}$ can be expressed as

$$B = \frac{B_o}{|\vec{r} - \vec{r}'|} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|},$$

where the second factor is the unit vector extending from the source to the observation point. Taking the dot product of B with the unit vectors in the radial and angular directions of the physical blockage cylindrical coordinate system yields

$$B_r = \frac{B_o}{|\vec{r} - \vec{r}'|} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|} \bullet \frac{\vec{r}}{r} = \frac{B_o}{|\vec{r} - \vec{r}'|^2}(r - r'\cos\theta) \tag{4}$$

and

$$B_\theta = \frac{B_o}{|\vec{r} - \vec{r}'|^2}(\vec{r} - \vec{r}') \bullet \theta = B_o \frac{r'\sin\theta}{|\vec{r} - \vec{r}'|^2} . \tag{5}$$

Expanding $\dfrac{1}{|\vec{r} - \vec{r}'|^2}$ in the binomial expansion yields

$$\frac{1}{|\vec{r} - \vec{r}'|^2} \approx \frac{1}{r^2}\left(1 - \frac{r'^2}{r^2} + \frac{2r'}{r}\cos\theta\right) \quad \text{for} \quad \vec{r} \geq \vec{r}' \tag{6}$$

and

$$\frac{1}{|\vec{r} - \vec{r}'|^2} \approx \frac{1}{r'^2}\left(1 - \frac{r^2}{r'^2} + \frac{2r}{r'}\cos\theta\right) \quad \text{for} \quad \vec{r} \leq \vec{r}' . \tag{7}$$

THE MATHEMATICAL SOLUTION OF THE B AND H FIELDS FOR THE $\dfrac{1}{|\vec{r} - \vec{r}'|}$ SOURCE

We substitute equations (6) and (7) into equations (4) and (5) and the result into the source terms of equations (1), (2), and (3). By applying the boundary conditions for the B and H fields to the resulting equations and setting the coefficients of the powers of the sines and cosines equal to zero, we generate the 10 equations and 10 unknowns and solve them using the *Mathematica* [7] software, as shown in the appendix of this report.

The boundary at $r = b$ has two cases: $\vec{r} \leq \vec{r}'$ and $\vec{r} \geq \vec{r}'$. For the continuity of B_r, the coefficients of $(\cos\theta)^0$ are

$$\frac{B_o}{r'^2}\left(b-\frac{b^3}{r'^2}\right)=\alpha_0\frac{1}{b^2}-\frac{3}{2}\alpha_2\frac{1}{b^4}-\mu\gamma_0\frac{1}{b^2}-\mu\beta_1-\mu\beta_2 b+\frac{3}{2}\mu\frac{\gamma_2}{b^4}\quad\text{for}\ \ \bar r\le\bar r'$$

(8)

and

$$\frac{B_o}{b^2}\left(b-\frac{r'^2}{b}\right)=\alpha_0\frac{1}{b^2}-\frac{3}{2}\alpha_2\frac{1}{b^4}-\mu\gamma_0\frac{1}{b^2}-\mu\beta_1-\mu\beta_2 b+\frac{3}{2}\mu\frac{\gamma_2}{b^4}\quad\text{for}\ \ \bar r\ge\bar r';$$

(9)

the coefficients of $(\cos\theta)^1$ are

$$\frac{B_0}{r'^2}\left(\frac{3b^2}{r'}-r'\right)=-2\alpha_1\frac{1}{b^3}-2\mu\gamma_1\frac{1}{b^3}\quad\text{for}\ \ \bar r\le\bar r'$$

(10)

and

$$\frac{B_0}{b^2}\left(\frac{r'^3}{b^2}+r'\right)=-2\alpha_1\frac{1}{b^3}-2\mu\gamma_1\frac{1}{b^3}\quad\text{for}\ \ \bar r\ge\bar r';$$

(11)

and the coefficients of $(\cos\theta)^2$ are

$$\frac{-2bB_o}{r'^2}=\frac{9}{2}\alpha_2\frac{1}{r^4}+3\mu\beta_2 b-\frac{9}{2}\mu\frac{\gamma_2}{b^4}\quad\text{for}\ \ \bar r\le\bar r'$$

(12)

and

$$\frac{-2r'^2 B_o}{b^3}=\frac{9}{2}\alpha_2\frac{1}{r^4}+3\mu\beta_2 b-\frac{9}{2}\mu\frac{\gamma_2}{b^4}\quad\text{for}\ \ \bar r\ge\bar r'.$$

(13)

For the continuity of H_θ, there are no terms in the coefficient for $(\cos\theta)^0$. For the coefficient of $(\sin)^1$,

$$\frac{B_0}{r'}\left(1-\frac{b^2}{r'^2}\right)-\frac{\alpha_1}{b^3}=-\beta_1-\frac{\gamma_1}{b^3}\quad\text{for}\ \ \bar r\le\bar r'$$

(14)

and

$$\frac{B_0}{b^2}\left(r'-\frac{r'^3}{b^2}\right)-\frac{\alpha_1}{b^3}=-\beta_1-\frac{\gamma_1}{b^3}\quad\text{for}\ \ \bar r\ge\bar r'.$$

(15)

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For the coefficient of $(\sin\theta\cos\theta)^1$,

$$\frac{2B_0 b}{r'^2}+\frac{3\alpha_2}{b^3}=3\beta_2 b^2+\frac{3\gamma_2}{b^4}\quad\text{for }\bar{r}\le\bar{r}' \tag{16}$$

and

$$\frac{2B_0 r'^2}{b^3}+\frac{3\alpha_2}{b^3}=3\beta_2 b^2+\frac{3\gamma_2}{b^4}\quad\text{for }\bar{r}. \tag{17}$$

For the continuity of B_r at $r=a$, the coefficient of $(\cos\theta)^0$ is

$$\frac{\mu\gamma_0}{a^2}-\mu\beta_2 a+\frac{3}{2}\frac{\gamma_2}{a^4}=-\frac{\delta_2 a}{2}\;; \tag{18}$$

the coefficient of $(\cos\theta)^1$ is

$$\mu\left(\beta_1-\frac{2\gamma_1}{a^3}\right)=\delta_1\;; \tag{19}$$

and the coefficient of $(\cos\theta)^2$ is

$$\mu\left(3\beta_2 a-\frac{9}{2}\frac{\gamma_2}{a^4}\right)=\frac{3}{2}\delta_2 a\;. \tag{20}$$

For the continuity of H_θ at $r=a$, the coefficient of $(\sin)^1$ is

$$\beta_1 a+\gamma_1\frac{1}{a^2}=\delta_1 a \tag{21}$$

and the coefficient of $(\sin\theta\cos\theta)^1$ is

$$\beta_2 a^2+\gamma_2\frac{1}{a^3}=\delta_2 a^2\;. \tag{22}$$

The radii a and b are fixed known numbers and the unknowns are $\alpha_0,\alpha_1,\alpha_2,\beta_1,\beta_2,\gamma_0,\gamma_1,\gamma_2,\delta_1$, and δ_2, with the following solutions:

$$\alpha_o = \frac{B_o\left(X_1\left(6\mu a^2 b - 12\mu^2 a^5 b + 6a^3 b^2 + 18\mu a^3 b^2 + 12\mu^2 a^3 b^2 + 4\mu b^5 + 6\mu^2 b^5\right)\right)}{6X_2 X_4 r'^4}$$

$$\frac{-6b^3 X_2 X_3 - 3\mu X_2 X_5 r' - 6\mu^2 X_2 X_5 r' - 6b X_2 X_3 r'^2 + 6b^4 X_4 X_6 r'^2\right)}{6X_2 X_4 r'^4},$$

$$\alpha_1 = \frac{B_o\left(-3b^2 + r'^2\right)}{2r'^3}, \qquad\qquad \alpha_2 = \frac{2B_o b^4 X_6}{3X_2 r'^2},$$

$$\gamma_o = \frac{B_o\left(1 - 3\mu + 2\mu^2\right)a^3 b^4 X_7}{\mu X_2 r'^2}, \qquad\qquad \gamma_1 = \frac{B_o\left(1 - \mu\right)a^3 X_5}{2X_4 r'^3},$$

$$\gamma_2 = \frac{2B_o\left(-1 + 2\mu\right)a^5 b^4 X_7}{3X_2 r'^2},$$

$$\delta_1 = \frac{-3B_o \mu X_5}{2X_2 r'^2}, \qquad\qquad \delta_2 = \frac{10B_o \mu b^4 X_7}{3X_2 r'^2},$$

which were solved numerically using the *Mathematica* software (see the appendix).

Substituting the above Legendre polynomial coefficients back into equations (1), (2), and (3) yields for $r > b$

$$\vec{H} = \hat{r}\left[\frac{B_0}{r^2 + r'^2 - 2rr'}(r - r'\cos\theta) - \frac{\alpha_1}{r^2}\cos\theta + \frac{\alpha_0}{r^2} + \frac{3\alpha_2}{2r^4}\left(3\cos^2\theta - 1\right)\right]$$

$$+ \hat{\theta}\left[\frac{B_0 r'\sin\theta}{r^2 + r'^2 - 2rr'} - \frac{\alpha_1}{r}\sin\theta + \frac{\alpha_2}{2r^4}\left(6\cos\theta\sin\theta\right)\right];$$

for $b > r > a$

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$$\bar{H} = \hat{r}\left[\left(\gamma_1 \frac{2}{r^3} - \beta_1\right)\cos\theta + \frac{\gamma_0}{r^2} + \left(\gamma_2 \frac{3}{r^4} + 2\beta_2 r\right)\left(3\cos^2\theta - 1\right)\right]$$

$$+ \hat{\theta}\left[\left(\beta_1 + \frac{\gamma_1}{r^3}\right)\sin\theta - \beta_2 r + \frac{6\gamma_2}{r^4}\sin\theta\cos\theta\right];$$

and for $r < b$

$$\bar{H} = \hat{r}\left[\frac{2\delta_1}{r^3}\cos\theta - \frac{3\delta_2}{4r^4}\left(3\cos^2\theta - 1\right) + \frac{\delta_0}{r^2}\right] + \hat{\theta}\left[\frac{\delta_1}{r^3}\sin\theta + \frac{3\delta_2}{r^4}\sin\theta\cos\theta\right].$$

THE MATHEMATICAL FORM FOR THE $\dfrac{1}{|\bar{r} - \bar{r}'|^3}$ SOURCE

The H fields are given in three regions: $r > b$, $a < r < b$, and $r < a$. For $r > b$, the H field is given by

$$\bar{H} = \frac{B_0}{|\bar{r} - \bar{r}'|^3}\frac{(\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|} - \nabla\sum_{\ell=0}^{2}\frac{\alpha_\ell}{r^{\ell+1}}P_\ell\left(\cos(\theta)\right); \tag{23}$$

for $a < r < b$,

$$\bar{H} = -\nabla\sum_{\ell=0}^{2}\left(\beta_\ell r^\ell + \frac{\gamma_\ell}{r^{\ell+1}}P_\ell\left(\cos(\theta)\right)\right); \tag{24}$$

and for $r < a$,

$$\bar{H} = -\nabla\sum_{\ell=0}^{2}\delta_\ell r^\ell P_\ell\left(\cos(\theta)\right). \tag{25}$$

Expanding equations (23), (24), and (25) yields for $r > b$

$$\bar{H} = \hat{r}\left[\frac{B_o}{\left(r^2 + r'^2 - 2rr'\right)^2}(r - r'\cos\theta) - \frac{\alpha_1}{r^2}\cos\theta + \frac{\alpha_0}{r^2} + \frac{3\alpha_2}{2r^4}\left(3\cos^2\theta - 1\right)\right]$$

$$+ \hat{\theta}\left[\frac{B_o r'\sin\theta}{\left(r^2 + r'^2 - 2rr'\right)^2} - \frac{\alpha_1}{r}\sin\theta + \frac{\alpha_2}{2r^4}\left(6\cos\theta\sin\theta\right)\right];$$

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for $b > r > a$,

$$\bar{H} = \hat{r}\left[\left(\gamma_1\frac{2}{r^3} - \beta_1\right)\cos\theta + \frac{\gamma_0}{r^2} + \left(\gamma_2\frac{3}{r^4} + 2\beta_2 r\right)\left(3\cos^2\theta - 1\right)\right]$$

$$+ \hat{\theta}\left[\left(\beta_1 + \frac{\gamma_1}{r^3}\right)\sin\theta - \beta_2 r + \frac{6\gamma_2}{r^4}\sin\theta\cos\theta\right];$$

and for $r < b$,

$$\bar{H} = \hat{r}\left[\frac{2\delta_1}{r^3}\cos\theta - \frac{3\delta_2}{4r^4}(3\cos^2\theta - 1) + \frac{\delta_0}{r^2}\right]$$

$$+ \hat{\theta}\left[\frac{\delta_1}{r^3}\sin\theta + \frac{3\delta_2}{r^4}\sin\theta\cos\theta\right].$$

THE $\frac{1}{|\bar{r} - \bar{r}'|^3}$ SOURCE IN COMPONENT FORM AND ITS BINOMIAL EXPANSION

The B field for the $\frac{1}{|\bar{r} - \bar{r}'|^3}$ can be expressed as

$$B = \frac{B_o}{|\bar{r} - \bar{r}'|^3}\frac{(\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|},$$

where the second factor is the unit vector extending from the source to the observation point. Taking the dot product of B with the unit vectors in the radial and angular directions of the physical blockage cylindrical coordinate system yields

$$B_r = \frac{B_o}{|\bar{r} - \bar{r}'|^3}\frac{(\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|} \cdot \frac{\bar{r}}{r} = \frac{B_o}{|\bar{r} - \bar{r}'|^4}(r - r'\cos\theta) \qquad (26)$$

and

$$B_\theta = \frac{B_o}{|\bar{r} - \bar{r}'|^4}(\bar{r} - \bar{r}') \cdot \theta = B_o\frac{r'\sin\theta}{|\bar{r} - \bar{r}'|^4}. \qquad (27)$$

Expanding $\frac{1}{|\bar{r} - \bar{r}'|^4}$ in the binomial expansion yields

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$$\frac{1}{|\bar{r}-\bar{r}'|^4} \approx \frac{1}{r^4}\left(1-2\left(\frac{r'^2}{r^2}-\frac{2r'}{r}\cos\theta\right)\right) \quad \text{for} \quad \bar{r}\geq\bar{r}' \tag{28}$$

and

$$\frac{1}{|\bar{r}-\bar{r}'|^4} \approx \frac{1}{r'^4}\left(1-2\left(\frac{r^2}{r'^2}-\frac{2r}{r'}\cos\theta\right)\right) \quad \text{for} \quad \bar{r}\leq\bar{r}'. \tag{29}$$

THE MATHEMATICAL SOLUTION OF THE B AND H FIELDS FOR THE $\frac{1}{|\bar{r}-\bar{r}'|^3}$ SOURCE

We substitute equations (28) and (29) into equations (26) and (27) and the result into the source terms of equations (23), (24), and (25). By applying the boundary conditions for the B and H fields to the resulting equations and setting the coefficients of the powers of the sines and cosines equal to zero, we generate the 10 equations and 10 unknowns, which are solved to obtain the coefficients of expansion. These coefficients are then substituted back into equations (1), (2), and (3) to yield a numerical solution.

The boundary at $r = b$ has two cases: $\bar{r}\leq\bar{r}'$ and $\bar{r}\geq\bar{r}'$. For the continuity of B_r, the coefficients of $(\cos\theta)^0$ are

$$\frac{B_0}{r'^3}\left(1-\frac{2b^2}{r'^2}\right)-\frac{\alpha_0}{b^2}+\frac{3}{2}\left(\frac{\alpha_2}{b^4}\right)=-\mu\gamma_0\left(\frac{1}{b^2}\right)-\mu\beta_2 b+\frac{3}{2}\left(\frac{\mu\gamma_2}{b^4}\right) \quad \text{for} \quad \bar{r}\leq\bar{r}' \tag{30}$$

and

$$\frac{B_0}{b^3}\left(1-\frac{2r'^2}{b^2}\right)-\frac{\alpha_0}{b^2}+\frac{3}{2}\left(\frac{\alpha_2}{b^4}\right)=-\mu\gamma_0\left(\frac{1}{b^2}\right)-\mu\beta_2 b+\frac{3}{2}\left(\frac{\mu\gamma_2}{b^4}\right) \quad \text{for} \quad \bar{r}\geq\bar{r}' \;; \tag{31}$$

the coefficients of $(\cos\theta)^1$ are

$$\frac{B_0}{r'^4}\left(\frac{4b^2}{r'}-b+\frac{2b^2}{r'}\right)-\frac{2\alpha_1}{b^3}=\mu\beta_1-\mu\gamma_1\left(\frac{2}{b^3}\right) \quad \text{for} \quad \bar{r}\leq\bar{r}' \tag{32}$$

and

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$$\frac{B_0}{b^4}\left(3r' + \frac{r'^3}{b^2}\right) - \frac{2\alpha_1}{b^3} = \mu\beta_1 - \mu\gamma_1\left(\frac{2}{b^3}\right) \quad \text{for} \quad \bar{r} \geq \bar{r}' \ ; \tag{33}$$

and the coefficients of $(\cos\theta)^2$ are

$$-\frac{B_0}{r'^4}(4b) - \frac{9\alpha_2}{2b^4} - = 3\frac{\mu}{2}\left(2\beta_2 b - \frac{3\gamma_2}{b^4}\right) \quad \text{for} \quad \bar{r} \leq \bar{r}' \tag{34}$$

and

$$-\frac{B_0}{b^5}(4r'^2) - \frac{9\alpha_2}{2b^4} - = 3\frac{\mu}{2}\left(2\beta_2 b - \frac{3\gamma_2}{b^4}\right) \quad \text{for} \quad \bar{r} \geq \bar{r}' \ . \tag{35}$$

For the continuity of H_θ at $r = b$, there are no terms in the coefficient for $(\cos\theta)^0$. For the coefficients of $(\sin)^1$,

$$\frac{B_0}{r'^4}\left(r' - \frac{2b^2}{r'}\right) - \frac{\alpha_1}{b^2} = -\left(\beta_1 b + \gamma_1\frac{1}{b^2}\right) \quad \text{for} \quad \bar{r} \leq \bar{r}' \tag{36}$$

and

$$\frac{B_0}{r^4}\left(r' - \frac{2r'^3}{b^2}\right) - \frac{\alpha_1}{b^2} = -\left(\beta_1 b + \gamma_1\frac{1}{b^2}\right) \quad \text{for} \quad \bar{r} \geq \bar{r}' \ . \tag{37}$$

For the coefficients of $(\sin\theta\cos\theta)^1$,

$$\frac{B_0}{r'^4}(4b) + 3\frac{\alpha_2}{b^3} = -\frac{3}{\gamma_2}\left(\frac{1}{b^3}\right) \quad \text{for} \quad \bar{r} \leq \bar{r}' \tag{38}$$

and

$$\frac{B_0}{b^5}(4r'^2) + 3\frac{\alpha_2}{b^3} = -\frac{3}{\gamma_2}\left(\frac{1}{b^3}\right) \quad \text{for} \quad \bar{r} \geq \bar{r}' \ . \tag{39}$$

For the continuity of B_r at $r = a$, the coefficient of $(\cos\theta)^0$ is

$$-\frac{\gamma_0}{a^2}\mu + \frac{1}{2}\left(2\beta_2 a - \gamma_2\frac{3}{a^4}\right) = 0 \ ; \tag{40}$$

the coefficient of $(\cos\theta)^1$ is

$$\beta_1 - \gamma_1 \frac{2}{a^3} = \delta_1 ; \tag{41}$$

and the coefficient of $(\cos\theta)^2$ is

$$\frac{3}{2}\left(2\beta_2 a - \gamma_2 \frac{3}{a^4}\right) = 3\delta_2 a . \tag{42}$$

For the continuity of H_θ at $r = a$, the coefficient of $(\sin)^1$ is

$$-\left(\beta_1 a + \gamma_1 \frac{1}{a^2}\right) = -\delta_1 a \tag{43}$$

and the coefficient of $(\sin\theta\cos\theta)^1$ is

$$\beta_2 a^2 + \gamma_2 \frac{1}{a^3} = \delta_2 a^2 . \tag{44}$$

THE MATHEMATICAL SOLUTION OF THE B AND H FIELDS FOR THE $\frac{1}{|\vec{r} - \vec{r}'|}$ CABLE SOURCE

The cable source is perpendicular to the radial field source and is of the same magnitude. The cable source field in the presence of the physical blockage was solved with the binomial and Legendre polynomial expansions in the same way that the solutions for the previous two sources presented in this report were obtained. The solution to the fields at $\theta = 0$ yields disturbance fields, which are functions of angular derivatives of the Legendre polynomials. The angular derivatives of the Legendre polynomials yield the angular component of the disturbance field, and each term has a $\sin\theta$ factor. Setting $\theta = 0$ yields zero for the disturbance field, which the experiments conducted at $\theta = 0$ proved to be true, as shown in figure 2.

The source field from the cable is given by

$$\vec{B}_w = \left(\frac{B_0}{|\vec{r} - \vec{r}'|} \right) \hat{\theta}_w ,$$

where $B_0 = \dfrac{I}{2\pi}$ (I is the current in the wire) and $\hat{\theta}_w$ is the unit vector in the angular direction.

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It can easily be seen that the radial field is perpendicular to the angular field so that

$$\vec{B}_w \bullet \vec{B} = 0 \ .$$

Rewriting $\vec{B}_w$ in the coordinate system of the physical blockage yields

$$\vec{B}_w = B_{wr}\hat{r} + B_{w\theta}\hat{\theta} \ ,$$

and rewriting $\vec{B}$ of the radial field in the coordinate system of the physical blockage yields

$$\vec{B} = B_r\hat{r} + B_\theta\hat{\theta} \ .$$

Taking the dot product of the two previous equations results in

$$B_{wr}B_r + B_{w\theta}B_\theta = 0 \ .$$

Since the sources are orthogonal fields, the fields around the physical blockage are orthogonal as well. (This property is also characteristic of potential flow where streamlines (velocity fields) and equipotential surfaces are orthogonal and the Cauchy-Riemann conditions are valid.) As a consequence, the final solution of the field with a radial source and physical blockage is perpendicular to the field with an angular source and physical blockage, resulting in

$$H_{wr}H_r + H_{w\theta}H_\theta = 0 \ .$$

Next, we obtain

$$\frac{H_{wr}}{H_{w\theta}} = -\frac{H_\theta}{H_r} \ .$$

Hence, the ratio of the components of the total fields from the angular source is equal to the negative of the ratio of the fields from the radial source.

The fields from the angular source were easier to measure in the laboratory. The ratio of their measured components was checked against the ratio of the components of the fields from the radial sources, which is easier to predict theoretically, as the model shows.

EXPERIMENTS

The experiment was set up to measure the B and H fields from a wire with its axis parallel to the cylindrical shell blockage. This configuration produced an angular field, which varied as $\frac{1}{|\vec{r} - \vec{r}'|}$ from the source. The results of the measurements taken at $\theta = 0$ and $\theta = \pi$ (figure 2) show a negligible amount of angular field disturbance, which decreased as the frequency decreased. The model predicts zero angular disturbance effects in the magnetostatic limit. Thus, the measurements are in good agreement with the model for this experimental configuration and these angles. The slight increase in the angular field disturbance as the frequency increased can be explained as an eddy current-Lenz law response of the physical blockage to the source field.

CONCLUSIONS AND RECOMMENDATIONS

The theoretical model for the angular field from a cable source predicted no physical distortion of the angular component magnetic field at $\theta = 0$ and $\theta = \pi$ with respect to the axis of the physical blockage (see figure 1). The distortion of the angular component of the magnetic field was calculated by taking the angular derivative of the Legendre polynomial expansion. After these derivatives were taken, each term in the Legendre polynomial expansion had a sin θ factor. Hence, the disturbance will vanish at $\theta = 0$ and $\theta = \pi$. Experimental measurements of the angular components of the magnetic field confirmed this behavior (see the experimental plots in figure 2).

The theoretical model also predicted a periodicity with higher harmonics in the disturbance field, and, in fact, the disturbance field will vanish altogether on certain axes and be maximized on others. Besides the angular disturbance caused by a physical blockage on the angular field, the radial disturbance can be predicted by taking the radial derivatives of the Legendre polynomial expansion.

The other models developed were for the vector radial field sources that varied in inverse proportion to the distance and cube of the distance. These sources, represented in the EMC code called IEMCADS as power supplies and transformers, are simplistically treated by the IEMCADS as scalars; however, to solve for the distortion effects of the physical blockage on the magnetic source fields, the sources had to be treated as vectors in order to apply the boundary conditions that require the angular component of the H field and the normal component of the B field to be continuous at the inner and outer
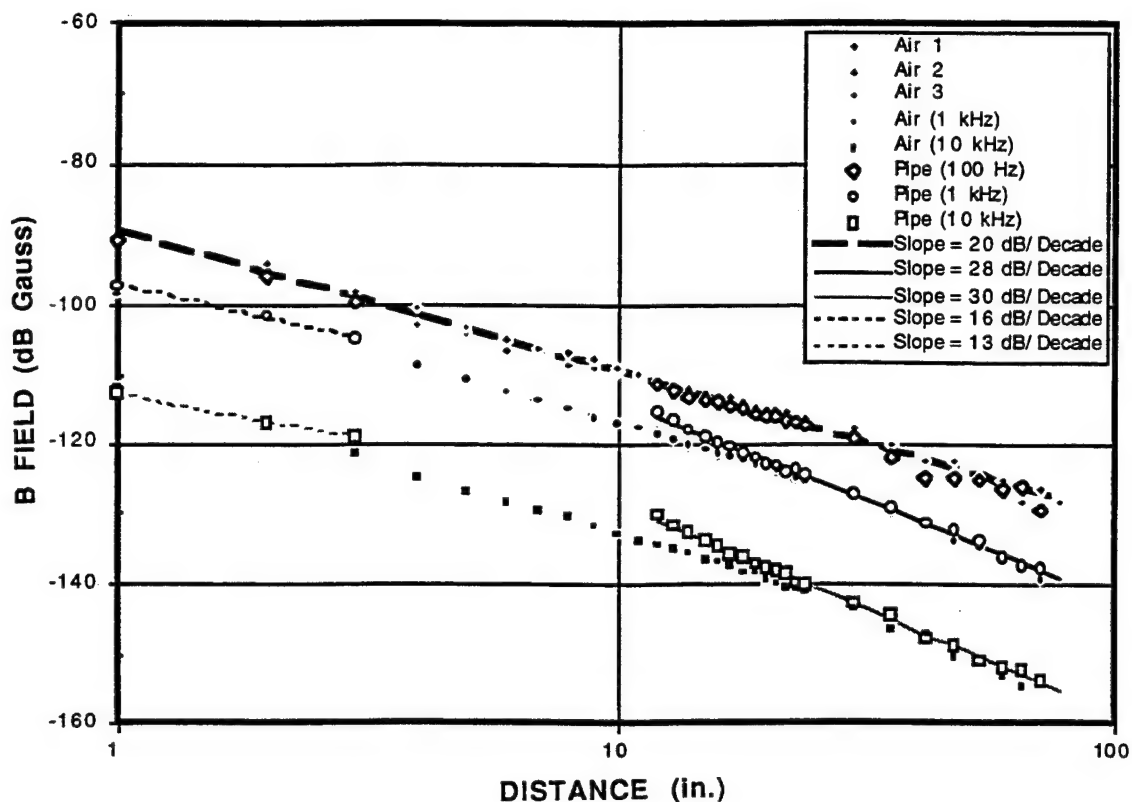
Figure 2. Measured Magnetic Field From a Straight Wire Near an Iron Pipe

interfaces. Upon application of the boundary conditions, a set of 10 equations and 10 unknowns was derived and solved to obtain the expansion coefficients for the Legendre polynomials, which resulted in the total field demonstrated in this report. Although these particular models were not checked in the laboratory since the mathematical sources are oversimplifications of real power supplies and transformers, they are an improvement over the mathematical sources used in IEMCADS because of their vector (as opposed to scalar) nature. Realistically, the nearfields from transformers and power supplies have both radial and angular components with coefficients in spherical coordinates. The physical blockage for the models assumes that the angular and spherical coordinates of the source were set equal to π, giving a worst-case situation when compared with other coordinates.

In summary, the objective of this investigation was to develop models to improve IEMCADS by incorporating the physical blockage. This meant keeping the source models as close to the IEMCADS models as possible. We could not, however, use the scalar fields designed to predict worst-case field conditions because the boundary conditions at the physical blockage could not be applied. Although the complexity of the

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nearfield complicated the process, we reconstructed sources in IEMCADS to give them a vector nature with more physical reality.

Future work should focus on the development of complete nearfield models for transformers and power supplies rather than on the approximate models developed here. The same techniques described earlier for determining the effects of physical blockage should be applied to these improved source models, and experiments to validate the models should be conducted. The source model used for the cables in this study was exact and requires no improvement. However, experimental measurements of the effects of the physical blockage at angles others than $\theta = 0$ and $\theta = \pi$ should be undertaken.

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APPENDIX

THE APPLICATION OF *MATHEMATICA* TO THE PROBLEM OF A CYLINDER IN A 1/*r* FIELD

◼ Description

This *Mathematica* workbook automates the solution of the model of a permeable cylinder in a 1/r field as defined by Dr. T. Anderson. It finds the coefficients of the Legendre Polynomials by solving for the boundary conditions.

◼ Defining Terms

```
u=1/r^2  (1-rPrime^2/r^2-(2*rPrime/r)*Cos[theta])
```

$$1 - \frac{rPrime^2}{r^2} - \frac{2 \; rPrime \; Cos[theta]}{r}$$
$$\frac{}{r^2}$$

```
v=(r+rPrime*Cos[theta])
```

r + rPrime Cos[theta]

```
w=rPrime*Sin[theta]
```

rPrime Sin[theta]

The number of terms in the Legendre Polynomials

```
n:=2
```

Φ outside the cylinder

```
Øm:=Sum[(alpha[l]/r^(l+1))*LegendreP[l,Cos[theta]],{l,0,n}]
```

Φ within the material of the cylinder

```
Øq:=Sum[(beta[l]*r^l  +  (gamma[l]/r^(l+1))) LegendreP[l,Cos[theta]],  {l,0,n}]
```

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Φ inside the cylinder

```
Øp:=Sum[delta[1]*r^1*LegendreP[1,Cos[theta]],   {1,0,n}]
```

■ Setting the Boundary conditions

```
FourEqn={
Bo*(v*u)-D[Øm,  r]-(-µ*D[Øq,  r])==0/.r->b,
Bo*(w*u)-(1/r)*D[Øm,  theta]-(-(1/r)*D[Øq,  theta])==0/.r->b,
-D[Øq,  r]-(-D[Øp,  r])==0/.r->a,
(-1/r)*D[Øq,  theta]-((-1/r)*D[Øp,  theta])==0/.r->a
}
```

$$\left\{\frac{\text{alpha}[0]}{b^2} + \frac{2\ \text{alpha}[1]\ \text{Cos}[\text{theta}]}{b^3} + \right.$$

$$\text{Bo}\ (b + \text{rPrime}\ \text{Cos}[\text{theta}])\ \left(1 - \frac{\text{rPrime}^2}{b^2} - \frac{2\ \text{rPrime}\ \text{Cos}[\text{theta}]}{b} + \right.$$

$$\left. \frac{2}{b^2} \right.$$

$$\frac{3\ \text{alpha}[2]\ (-1 + 3\ \text{Cos}[\text{theta}]^2)}{2\ b^4} + \mu$$

$$\left(-\left(\frac{\text{gamma}[0]}{b^2}\right) + \text{Cos}[\text{theta}]\ (\text{beta}[1] - \frac{2\ \text{gamma}[1]}{b^3}) + \right.$$

```
                        2              3 gamma[2]
      (-1 + 3 Cos[theta] ) (2 b beta[2] - ---------)
                                              4
                                             b
      --------------------------------------------------) == 0,
                            2

              rPrime   2 rPrime Cos[theta]
Bo rPrime (1 - ------ - ------------------) Sin[theta]
                2              b
               b
------------------------------------------------------ -
                      2
                     b

    alpha[1] Sin[theta]     3 alpha[2] Cos[theta] Sin[theta]
-(- ------------------- ) - -------------------------------- +
            2                              3
           b                              b

                  gamma[1]                                   2              gamma[2]
  -((b beta[1] + --------) Sin[theta]) - 3 Cos[theta] (b beta[2] + --------) Sin[theta]
                    2                                                   3
                   b                                                   b
  ------------------------------------------------------------------------------------- == 0
                                       b

                                        2              gamma[0]
, Cos[theta] delta[1] + a (-1 + 3 Cos[theta] ) delta[2] + --------
                                                             2
                                                            a
```

$$\mathrm{Cos[theta]}\ \left(\mathrm{beta[1]} - \frac{2\ \mathrm{gamma[1]}}{a^3} - \frac{(-1 + 3\ \mathrm{Cos[theta]}^2)\ \left(2\ a\ \mathrm{beta[2]} - \dfrac{3\ \mathrm{gamma[2]}}{a^4}\right)}{a^2}\right) == 0,$$

$$-(a\ \mathrm{delta[1]}\ \mathrm{Sin[theta]}) - \frac{3\ a^2\ \mathrm{Cos[theta]}\ \mathrm{delta[2]}\ \mathrm{Sin[theta]}}{a} -$$

$$\frac{-\left(\left(a\ \mathrm{beta[1]} + \dfrac{\mathrm{gamma[1]}}{a^2}\right)\mathrm{Sin[theta]}\right) - 3\ \mathrm{Cos[theta]}^2\ \left(a^2\ \mathrm{beta[2]} + \dfrac{\mathrm{gamma[2]}}{a^3}\right)\mathrm{Sin[theta]}}{a} == 0$$

}

Simplify the result from above without using Trigonometric identities

FourEqn1=Simplify[FourEqn,Trig->False]

$$\left\{\frac{\mathrm{alpha[0]}}{b^2} + \frac{2\ \mathrm{alpha[1]}\ \mathrm{Cos[theta]}}{b^3} + \right.$$

$$\left. \frac{\mathrm{Bo}\ (b + \mathrm{rPrime}\ \mathrm{Cos[theta]})\ (b^2 - \mathrm{rPrime}^2 - 2\ b\ \mathrm{rPrime}\ \mathrm{Cos[theta]})}{b^4} + \right.$$

```
  3 alpha[2] (-1 + 3 Cos[theta] )
- -------------------------------- + μ
                 4
              2 b


    gamma[0]                          2 gamma[1]
(-(--------) + Cos[theta] (beta[1] - ----------) +
       2                                 3
      b                                 b

                       2                3 gamma[2]
 (-1 + 3 Cos[theta] ) (2 b beta[2] - ----------)
                                         4
                                        b
 ------------------------------------------------) == 0,
                        2


  2                         3              4
((b  Bo rPrime - Bo rPrime  + b alpha[1] - b  beta[1] - 2 b Bo rPrime  Cos[theta] +

                        5                                       2
 3 alpha[2] Cos[theta] - 3 b  beta[2] Cos[theta] - b gamma[1] - 3 Cos[theta] gamma[2])

        4                2                          2
Sin[theta]) / b  == 0, Cos[theta] delta[1] + a (-1 + 3 Cos[theta] ) delta[2] +

                                                                     gamma[0]
                                                                     --------
                                                                        2
       2                          2                                    a
(-1 + 3 Cos[theta] ) (2 a beta[2] - 3 gamma[2]
                                    ----------
                                        4
                                       a
------------------------------------------------- == 0,
                        2


                          2 gamma[1]
Cos[theta] (beta[1] - ----------) -
                          3
                         a
```

$$((a^4\,\beta[1] + 3\,a^5\,\beta[2]\,\cos[\theta] - a^4\,\delta[1] - 3\,a^5\,\cos[\theta]\,\delta[2] +$$

$$a\,\gamma[1] + 3\,\cos[\theta]\,\gamma[2])\,\sin[\theta]) / a^4 == 0\}$$

Find all the coefficients of Sin(θ) and Cos(θ)

```
frst=Table[CoefficientList[First[FourEqn1[[i]]],{Cos[theta],Sin[theta]}],  {i,1,4}]
```

```
       Bo rPrime   alpha[0]   3 alpha[2]              μ gamma[0]   3 μ gamma[2]
{{{Bo - --------- + -------- - ---------- - μ b beta[2] - ---------- + ------------},
           b^3        b^2        2 b^4                      b^2          2 b^4

    Bo rPrime   Bo rPrime   2 alpha[1]              2 gamma[1]
  {-(---------) - --------- + ---------- + μ (beta[1] - ----------)},
      b^2          b^4          b^3                      b^3

   -2 Bo rPrime   9 alpha[2]              9 μ gamma[2]
  {------------ + ---------- + 3 μ b beta[2] - ------------}},
       b^3          2 b^4                        2 b^4

       Bo rPrime   alpha[1]              gamma[1]
  {{0, --------- + -------- - beta[1] - --------},
          b^2        b^4                  b^3
```

$$\{0,\ \frac{-2\ Bo\ rPrime^2}{b^3} + \frac{3\ alpha[2]}{b^4} - 3\ b\ beta[2] - \frac{3\ gamma[2]}{b^4}\}\},$$

$$\{\{a\ beta[2] - a\ delta[2] + \frac{gamma[0]}{a^2} - \frac{3\ gamma[2]}{2\ a^4}\},\ \{-beta[1] + delta[1] + \frac{2\ gamma[1]}{a^3}\},$$

$$\{-3\ a\ beta[2] + 3\ a\ delta[2] + \frac{9\ gamma[2]}{2\ a^4}\}\},$$

$$\{\{0,\ beta[1] - delta[1] + \frac{gamma[1]}{a^3}\},\ \{0,\ 3\ a\ beta[2] - 3\ a\ delta[2] + \frac{3\ gamma[2]}{a^4}\}\}\}$$

Length[Flatten[frst]]

14

Remove all the coefficients that are equal to 0

frst=Complement[Flatten[frst], {0}]

$$\{beta[1] - delta[1] + \frac{gamma[1]}{a^3},\ -beta[1] + delta[1] + \frac{2\ gamma[1]}{a^3},$$

$$\frac{Bo\ rPrime}{b^2} - \frac{Bo\ rPrime}{b^4} + \frac{alpha[1]}{b^3} - beta[1] - \frac{gamma[1]}{b^3},$$

```
      Bo rPrime      Bo rPrime 3   2 alpha[1]                2 gamma[1]
   -(---------)  -  ----------- + ---------- + μ (beta[1] -  ----------),
        2               4             3                          3
        b               b             b                          b

                          gamma[0]   3 gamma[2]
  a beta[2] - a delta[2] + -------- - ----------, 3 a beta[2] - 3 a delta[2] +
                            2           4
                           a          2 a

                                                       3 gamma[2]
                                                       ----------,
                                                           4
                                                          a

                              9 gamma[2]
  -3 a beta[2] + 3 a delta[2] + ----------,
                                 4
                                2 a

  -2 Bo rPrime   3 alpha[2]                3 gamma[2]
  ------------ + ---------- - 3 b beta[2] - ----------,
       3            4                           4
       b            b                           b

  -2 Bo rPrime   9 alpha[2]                  9 μ gamma[2]
  ------------ + ---------- + 3 μ b beta[2] - -----------,
       3            4                            4
       b           2 b                          2 b

  Bo   Bo rPrime   alpha[0]   3 alpha[2]                μ gamma[0]   3 μ gamma[2]
  -- - --------- + -------- - ---------- - μ b beta[2] - ---------- + ------------}
  b        3          2           4                          2            4
           b          b           b                         b           2 b
```

Length[Flatten[first]]

10
```
```

Set all the coefficients equal to 0

Equations=Table[Simplify[Equal[frst[[i]],0]],{i,1,Length[frst]}]

TableForm[Equations]

$$\text{beta}[1] - \text{delta}[1] + \frac{\text{gamma}[1]}{a^3} == 0$$

$$-\text{beta}[1] + \text{delta}[1] + \frac{2\,\text{gamma}[1]}{a^3} == 0$$

$$\frac{b^2\,\text{Bo}\,\text{rPrime} - \text{Bo}\,\text{rPrime} + b\,\text{alpha}[1] - b^3\,\text{beta}[1] - b\,\text{gamma}[1]}{b^4} == 0$$

$$-\frac{b^2\,(\text{Bo}\,\text{rPrime}) - \text{Bo}\,\text{rPrime} + 2\,b\,\text{alpha}[1] + \mu\,b^4\,\text{beta}[1] - 2\,\mu\,b\,\text{gamma}[1]}{b^4} == 0$$

$$a\,\text{beta}[2] - a\,\text{delta}[2] + \frac{\text{gamma}[0]}{a^2} + \frac{3\,\text{gamma}[2]}{2\,a^4} == 0$$

$$3\,a\,\text{beta}[2] - 3\,a\,\text{delta}[2] + \frac{3\,\text{gamma}[2]}{a^4} == 0$$

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$$-3\ a\ beta[2] + 3\ a\ delta[2] + \frac{9\ gamma[2]}{2\ a^4} == 0$$

$$\frac{-2\ b^2\ Bo\ rPrime + 3\ alpha[2] - 3\ b^5\ beta[2] - 3\ gamma[2]}{b^4} == 0$$

$$\frac{-4\ b^2\ Bo\ rPrime + 9\ alpha[2] + 6\ \mu\ b^5\ beta[2] - 9\ \mu\ gamma[2]}{2\ b^4} == 0$$

$$\frac{(2\ b^3\ Bo - 2\ b\ Bo\ rPrime^2 + 2\ b^2\ alpha[0] - 3\ alpha[2] - 2\ \mu\ b^5\ beta[2] - 2\ \mu\ b^2\ gamma[0] + 3\ \mu\ gamma[2])}{(2\ b^4)} == 0$$

Define all the possible unknowns

UnKnowns=Flatten[Table[{alpha[i],beta[i],gamma[i],delta[i]}, {i,0,n}]]

{alpha[0], beta[0], gamma[0], delta[0], alpha[1], beta[1], gamma[1], delta[1], alpha[2],

beta[2], gamma[2], delta[2]}

■ Solve the Equations for the Unknowns

Solve the equations for the unknowns

Coeffs=Solve[Equations, UnKnowns]

TableForm[Flatten[Coeffs]]

Simplify the final equations for the coefficients of the Legendre Polynomials

`simplify[Coeffs]`

$$\{\{alpha[0] \rightarrow \frac{Bo\ (-3\ b^2 + 5\ rPrime^2)}{3\ b},\quad gamma[0] \rightarrow 0,$$

$$alpha[1] \rightarrow \frac{Bo\ rPrime\ (b^2 - \mu\ b^2 + rPrime^2 + \mu\ rPrime^2)}{2\ b + \mu\ b},$$

$$delta[1] \rightarrow \frac{Bo\ rPrime\ (3\ b^2 - rPrime^2)}{(2 + \mu)\ b^4},\quad alpha[2] \rightarrow \frac{4\ (1 + \mu)\ b\ Bo\ rPrime^2}{9 + 6\ \mu},$$

$$delta[2] \rightarrow \frac{-2\ Bo\ rPrime^2}{3\ (3 + 2\ \mu)\ b^4},\quad beta[1] \rightarrow \frac{Bo\ rPrime\ (3\ b^2 - rPrime^2)}{(2 + \mu)\ b^4},\quad gamma[1] \rightarrow 0,$$

$$beta[2] \rightarrow \frac{-2\ Bo\ rPrime^2}{3\ (3 + 2\ \mu)\ b^4},\quad gamma[2] \rightarrow 0\}\}$$

■ Solving for H

Needs["Calculus`VectorAnalysis`"]

The Field outside the Cylinder:

```
H={0,(Bo/Abs[r-rPrime]*(r-rPrime/Abs[r-rPrime])),0}-
Grad[Sum[alpha[i]/r^i*LegendreP[i, Cos[theta]], {i, 0,2}],Cylindrical]
```

$$
\left\{ \frac{\text{alpha}[1]\ \text{Cos}[\text{theta}]}{r^2} + \frac{\text{alpha}[2]\ (-1 + 3\ \text{Cos}[\text{theta}]^2)}{r^3}, \right.
$$

$$
-\left(\frac{\text{alpha}[1]\ \text{Sin}[\text{theta}]}{r^2} \right) - \frac{3\ \text{alpha}[2]\ \text{Cos}[\text{theta}]\ \text{Sin}[\text{theta}]}{r^2}
$$

$$
\text{Bo}\ \left(r - \frac{\text{rPrime}}{\text{Abs}[r - \text{rPrime}]} \right)
$$

$$
\left. \frac{}{\text{Abs}[r - \text{rPrime}]}, 0 \right\}
$$

Substitute the Coefficients

H/.Coeffs

```
                   2     2         2
{{  Bo rPrime (b  - μ b  + rPrime  + μ rPrime ) Cos[theta]
    ----------------------------------------------------- +
                              2
                    (2 b + μ b) r

                          2
    4 (1 + μ) b Bo rPrime  (-1 + 3 Cos[theta] )
    ------------------------------------------- ,
                        3
              (9 + 6 μ) r

            rPrime
    Bo (r - ----------)
            Abs[r - rPrime]                    2     2         2
    -------------------------- - (-(Bo rPrime (b  - μ b  + rPrime  + μ rPrime ) Sin[theta]) -
         Abs[r - rPrime]                                         2
                                                       (2 b + μ b) r

                            2
    12 (1 + μ) b Bo rPrime  Cos[theta] Sin[theta]
    ---------------------------------------------) / r, 0}}
                        2
              (9 + 6 μ) r
```

Along the axis:

%/.theta->0

```
                       2                    2     2         2
{{  8 (1 + μ) b Bo rPrime      Bo rPrime (b  - μ b  + rPrime  + μ rPrime )
    ---------------------- + ---------------------------------------------- ,
                3                              2
      (9 + 6 μ) r                    (2 b + μ b) r

            rPrime
    Bo (r - ----------)
```

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$$\frac{\text{Abs}[r - \text{rPrime}]}{\text{Abs}[r - \text{rPrime}]}, \ 0\}\}$$

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